DS 574 Algorithmic Mechanism Design	**Lecture #19**
Fall 2023	Prof. Kira Goldner

Time-Inconsistent Planning: Present Bias [1, 3]

Today we'll talk about time-Inconsistent behavior, or present bias. The Noble Laureate, the economist George Akerlof, tells the following story: at the time he was in India and he needed to send a package to a friend of his, another economist named Joseph Stiglitz. Since he was in India sending the package was a bit of a hassle. So everyday, he woke up in the morning and decided that he will send the package the next day and then the next day and so on. At some point he realized he was behaving irrationally, but still he couldn't bring himself to send the package. Eventually, this story ended when a friend of his offered to send the package for him.

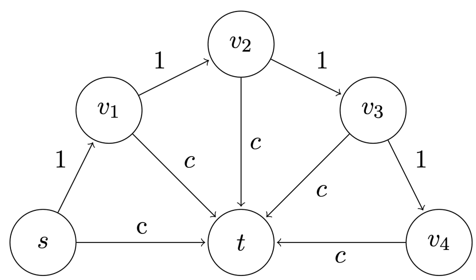

Figure 1: The fan graph for Akerlof's story.

Formally:

- Sending the package has a fixed cost c.
- There is a loss of use cost 1 for each day in which the package cannot be used.
- Total cost for sending on day t is: $c + t$.

The rational behavior is to send the package on the first day to minimize the total cost. *Present bias* [Akerlof] indicates that you perceive the cost of doing something today as inflated by some bias factor b. Thus: the cost of sending the package today is $b \cdot c$ and if $b \cdot c > b + c$ it is better to send it on the next day.

More generally, we define the model as follows:

1. There is a directed acyclic graph G with a source s and a target t.
2. Each edge e corresponds to some task and has a cost which captures the effort required for completing the task.

3. The agent needs to take a path from s to t. At each node v it will choose the $v-t$ path which is the shortest path in a graph in which the costs of all outgoing edges from v are multiplied by a factor of b.

This simple model is based on more elaborate model (quasi hyperbolic discounting). Formally:

Definition 1 (traversal)**.** An agent currently at v_i will continue to a node $v_{i+1} \in \arg\min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u,t)$. We refer to $C(v_i) = \min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u,t)$ as the perceived cost of agent i at v_i.

Let's see another example:

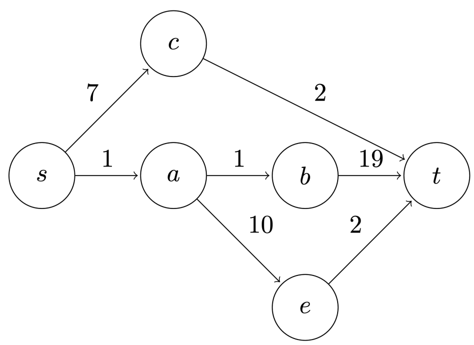

Figure 2: An example for graph traversal of present bias with $b = 2$.

Question: Consider an agent with present bias $b = 2$. Which path will he traverse in the graph in Figure 2?

The agent will take $s \to a \to b \to t$.

Choice Reduction and Its Benefits

In an experiment in a course at MIT: Students need to submit 3 assignments throughout the semester. In the beginning of the semester, each student was asked to set a deadline for each assignment. What is the rational behavior? A: Set all deadlines on the last day of classes.

What would you do?

In the experiment: Only 27% of the students chose to submit all assignments on the last day. Many students spaced out the deadlines and in particular it was shown that students with deadlines did better than students without and that students with enforced, equally-spaced deadlines did even better.

What does this tell us?

1. The setting deadlines helps.
2. Maybe some students are aware of their present bias. (We'll elaborate on this later.)

Consider a three week course in which the students need to complete two tasks. The cost of completing a single task in a week is 4, the cost for completing both in the same week is 9, and the cost of a week of studying without doing any tasks is 1. The task graph in Figure 3 models this scenario. In the graph, node $v_{i,j}$ corresponds to completing j tasks by week i.

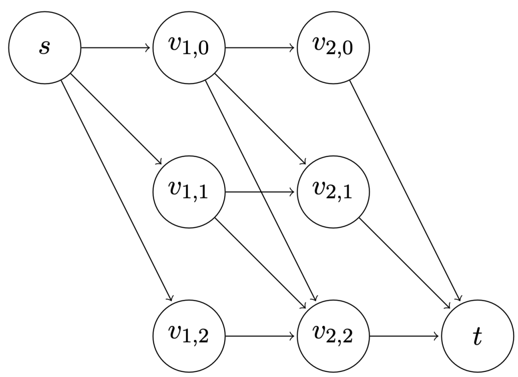

Figure 3: An example featuring the benefits of setting deadlines. Horizontally: weeks. Vertically: tasks. Horizontal edges 1, diagonal by 1 are 4, diagonal by 2 are 9.

Now, assume that there is a reward $R = 17$ for completing the course (reaching t) and the agent will traverse the graph as long as its perceived cost is less than R. How will an agent with present bias $b = 2$ traverse the graph?

$s \to v_{1,0} \to v_{2,0}$ and then drop out.

How can we help the student complete the course? Consider setting a deadline for the first assignment: the first assignment should be submitted by the second week. This means that in graph we delete the node $v_{2,0}$. What will the agent do now?

$s \to v_{1,0} \to t$.

This leads to the following algorithmic question: given a graph in which the agent does not reach t can we delete nodes and edges such that agent will reach t?

One way for approaching this question is hoping that if there is a traversable subgraph then there is always a traversable subgraph which is just a path. What do you think, is this true?

Here is a simple example showing that this isn't the case: In this example we have that

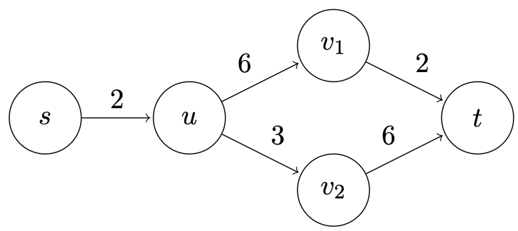

Figure 4: An example showing that the minimal traversable subgraph might not be a path.

$b = 2$ and $R = 12$. The graph is traversable, but if we remove v_1 the agent does not start traversing the graph and if we will remove v_2 the agent will take the first edge and then stop traversing the graph.

In general, the minimal traversable subgraph has a very specific structure. It is composed of P—the path that the agent will actually take. In addition, from each node of the path P there is at most a single path that crosses P once. We call this path a shortcut. These are paths that the agent plans to take but then it doesn't take them.

Research Directions:

- Cost ratio: quantifying how much present-biased agents lose due to their bias.
- Characterizing graph structures that lend themselves to bounded or exponential cost ratios.
- Sophisticated agents aware of their present bias [4].
- Additional work: principal-agent model [7], with sunk costs [5].
- Type 1 vs. type 2 utility [2].

Obviously Strategy Proof [6]

We need a few more standard game-theoretic definitions before we can understand this concept.

Definition 2. A k-player finite *extensive-form* game is defined by a finite, rooted tree T . Each node in T represents a possible state in the game, with leaves representing terminal states. Each internal (nonleaf) node v in T is associated with one of the players, indicating

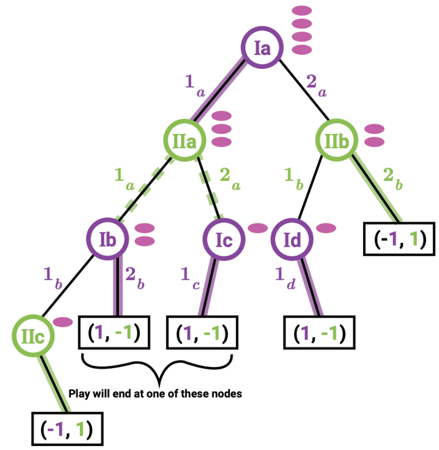

Figure 5: The Subtraction Game: Starting with a pile of four chips, two players alternate taking one or two chips. Player I goes first. The player who removes the last chip wins.

that it is his turn to play if/when v is reached. The edges from an internal node to its children are labeled with actions, the possible moves the corresponding player can choose from when the game reaches that state. Each leaf/terminal state results in a certain payoff for each player. A pure strategy for a player in an extensive-form game specifies an action to be taken at each of that player's nodes. A mixed strategy is a probability distribution over pure strategies.

Definition 3. Given an extensive-form game, the *normal form* of the game is the matrix of possible pure strategies and their resulting payoffs.

Sealed-bid second-price auction and ascending English auction have the same normal form, but not the same extensive form. In practice, people play them quite differently.

Earliest Point of Departure: Nodes I_i are in the *information set* $\alpha(S_i^1, S_i^2)$ iff

- $S_i^1 \neq S_i^2$ at I_i and
- I_i could have been reached by playing either S_i^1 or S_i^2.

Let $u_i^G(h, S_i, S_{-i}, v_i)$ be the utility to agent i in game G as a function of starting from history h with play proceeding according to S_i, S_{-i} and the resulting outcome evaluated according to preferences v_i.

Definition 4. A strategy S_i is *weakly dominant* if for all deviating strategies S_i' and other bidder strategies S_{-i},

$$u_i^G(h_0, S_i, S_{-i}, v_i) \geq u_i^G(h_0, S_i', S_{-i}, v_i).$$

Definition 5. A strategy S_i is *obviously dominant* if for all deviating strategies S_i' and nodes in the earliest point of departure $I_i \in \alpha(S_i, S_i')$:

$$\inf_{h \in I_i, S_{-i}} u_i^G(h, S_i, S_{-i}, v_i) \geq \sup_{h \in I_i, S_{-i}} u_i^G(h, S_i', S_{-i}, v_i).$$

Definition 6. A mechanism is *obviously strategyproof* if truth-telling is an obviously dominant strategy.

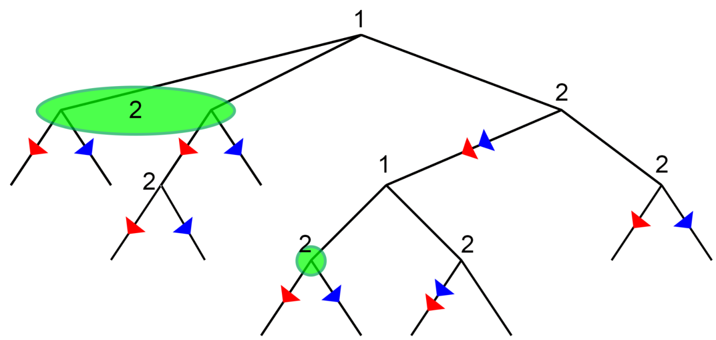

Figure 6: A depiction of the earliest point of departure for the red and blue strategies.

Consider when bidder 2's value is \$3, the two strategies of bidding truthfully and bidding \$5 (that is, dropping out at this value in the ascending auction). What history is *worst* for truth-telling and *best* for the deviating strategy? What is bidder 2's utility in each case? If bidder 2's utility is higher for truth-telling, then truth-telling is obviously dominant.

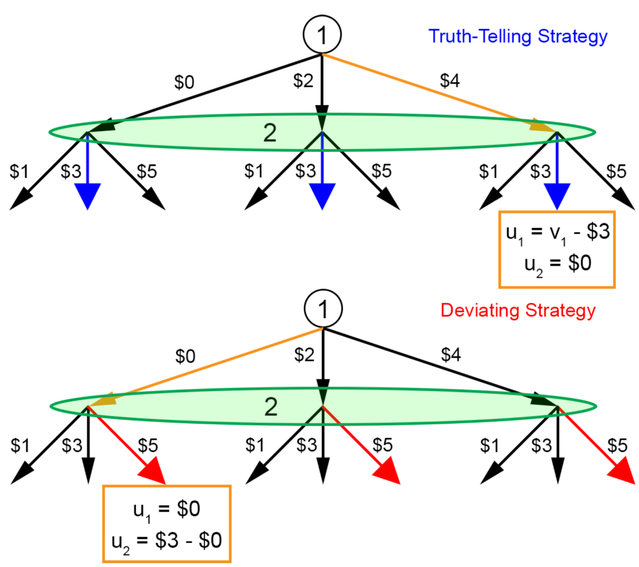

Figure 7: In the second-price sealed bid auction, the best history for the deviation of \$5 (bidder 1 at \$0) yields utility of \$3, whereas the worst history for truth-telling (bidder 1 at \$4) yields \$0 utility.

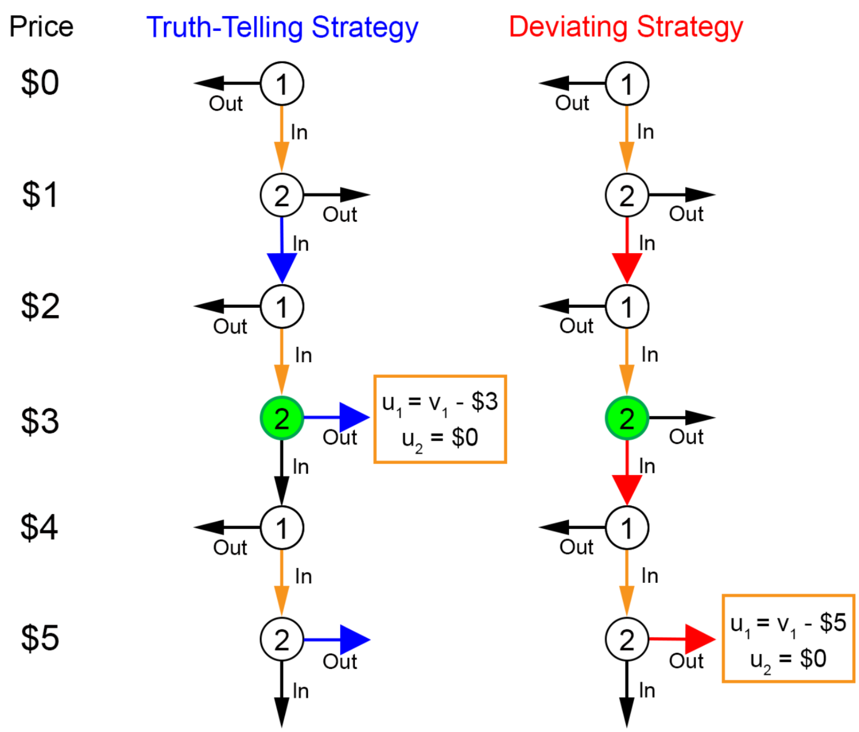

Figure 8: In the ascending English auction, the best history for the deviation of $5 (bidder 1 at >$5 is best, given that bidder 2 has gotten to $5) yields utility of $0, which is no better than worst history for truth-telling (bidder 1 at $4) which yields $0 utility.

Note the following:

- A strategy is a complete contingent plan of action.
- Weak dominance depends only on normal form.
- Obvious dominance depends on extensive form.
- The standard revelation principle does not apply.

Acknowledgements

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