

## Time-Inconsistent Planning: Present Bias

Story about Nobel Laureate George Akerlof who needed to mail a package to his friend, Joseph Stiglitz.

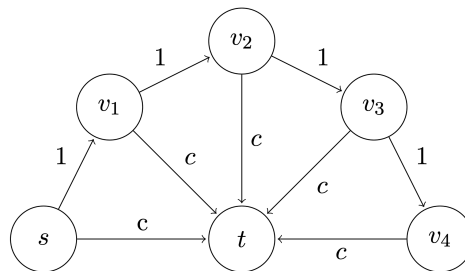


Figure 1: The fan graph for Akerlof's story.

Formally:

- Sending the package has a fixed cost  $c$ .
- There is a loss of use cost 1 for each day in which the package cannot be used.
- Total cost for sending on day  $t$  is:  $c + t$ .

The rational behavior is to send the package on the first day to minimize the total cost. *Present bias* [Akerlof] indicates that you perceive the cost of doing something today as inflated by some bias factor  $b$ . Thus:

More generally, we define the model as follows:

1. There is a directed acyclic graph  $G$  with a source  $s$  and a target  $t$ .
2. Each edge  $e$  corresponds to some task and has a cost which captures the effort required for completing the task.
3. The agent needs to take a path from  $s$  to  $t$ . At each node  $v$  it will choose the  $v-t$  path which is the shortest path in a graph in which the costs of all outgoing edges from  $v$  are multiplied by a factor of  $b$ .

This simple model is based on more elaborate model (quasi-hyperbolic discounting). Formally:

**Definition 1** (traversal). An agent currently at  $v_i$  will continue to a node  $v_{i+1} \in \arg \min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u, t)$ . We refer to  $C(v_i) = \min_{u \in N(v_i)} b \cdot c(v_i, u) + d(u, t)$  as the perceived cost of agent  $i$  at  $v_i$ .

Let's see another example:

**Question:** Consider an agent with present bias  $b = 2$ . Which path will he traverse in the graph in Figure 2?

## Choice Reduction and Its Benefits

Experiment in a course at MIT: Students need to submit 3 assignments throughout the semester. In the beginning of the semester, each student was asked to set a deadline for each assignment. What is the rational behavior?

What would you do?

In the experiment:

What does this tell us?

**Example:** 3 week course, 2 task. The cost of completing a single task in a week is 4. The cost for completing both in the same week is 9. The cost of a week of studying without doing any tasks is 1. The task graph in Figure 2 models this scenario. In the graph, node  $v_{i,j}$  corresponds to completing  $j$  tasks by week  $i$ .

Now, assume that there is a reward  $R = 17$  for completing the course (reaching  $t$ ) and the agent will traverse the graph as long as its perceived cost is less than  $R$ . How will an agent with present bias  $b = 2$  traverse the graph?

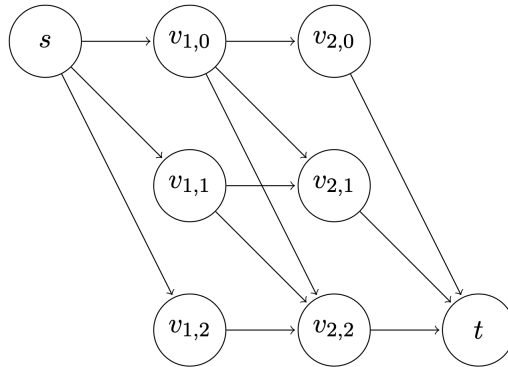


Figure 2: An example featuring the benefits of setting deadlines. Horizontally: weeks. Vertically: tasks.

How can we help the student complete the course? Consider setting a deadline for the first assignment: the first assignment should be submitted by the second week. This means that in graph we delete the node  $v_{2,0}$ . What will the agent do now?

This leads to the following algorithmic question: given a graph in which the agent does not reach  $t$  can we delete nodes and edges such that agent will reach  $t$ ?

One way for approaching this question is hoping that if there is a traversable subgraph then there is always a traversable subgraph which is just a path. Is this true?

### Research Directions:

- Cost ratio: quantifying how much present-biased agents lose due to their bias.

- Characterizing graph structures that lend themselves to bounded or exponential cost ratios.
- Sophisticated agents aware of their present bias.

## Obviously Strategy Proof

We need a few more standard game-theoretic definitions before we can understand this concept.

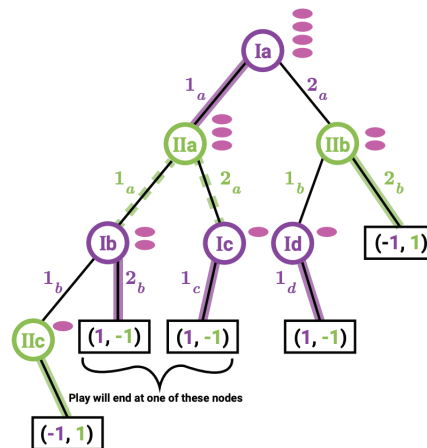


Figure 3: The Subtraction Game: Starting with a pile of four chips, two players alternate taking one or two chips. Player I goes first. The player who removes the last chip wins.

**Definition 2.** A  $k$ -player finite *extensive-form* game is defined by a finite, rooted tree  $T$ . Each node in  $T$  represents a possible state in the game, with leaves representing terminal states. Each internal (nonleaf) node  $v$  in  $T$  is associated with one of the players, indicating that it is his turn to play if/when  $v$  is reached. The edges from an internal node to its children are labeled with actions, the possible moves the corresponding player can choose from when the game reaches that state. Each leaf/terminal state results in a certain payoff for each player. A pure strategy for a player in an extensive-form game specifies an action to be taken at each of that player's nodes. A mixed strategy is a probability distribution over pure strategies.

**Definition 3.** Given an extensive-form game, the *normal form* of the game is the matrix of possible pure strategies and their resulting payoffs.

Sealed-bid second-price auction and ascending English auction have the same normal form, but not the same extensive form. In practice, people play them quite differently.

**Earliest Point of Departure:** Nodes  $I_i$  are in the *information set*  $\alpha(S_i^1, S_i^2)$  if and only if

- $S_i^1 \neq S_i^2$  at  $I_i$  and
- $I_i$  could have been reached by playing either  $S_i^1$  or  $S_i^2$ .

Let  $u_i^G(h, S_i, S_{-i}, v_i)$  be the utility to agent  $i$  in game  $G$  as a function of starting from history  $h$  with play proceeding according to  $S_i, S_{-i}$  and the resulting outcome evaluated according to preferences  $v_i$ .

**Definition 4.** A strategy  $S_i$  is *weakly dominant* if for all deviating strategies  $S'_i$  and other bidder strategies  $S_{-i}$ ,  $u_i^G(h_0, S_i, S_{-i}, v_i) \geq u_i^G(h_0, S'_i, S_{-i}, v_i)$

**Definition 5.** A strategy  $S_i$  is *obviously dominant* if for all deviating strategies  $S'_i$  and nodes in the earliest point of departure  $I_i \in \alpha(S_i, S'_i)$ :  $\inf_{h \in I_i, S_{-i}} u_i^G(h, S_i, S_{-i}, v_i) \geq \sup_{h \in I_i, S_{-i}} u_i^G(h, S'_i, S_{-i}, v_i)$

**Definition 6.** A mechanism is *obviously strategyproof* if truth-telling is an obviously dominant strategy.

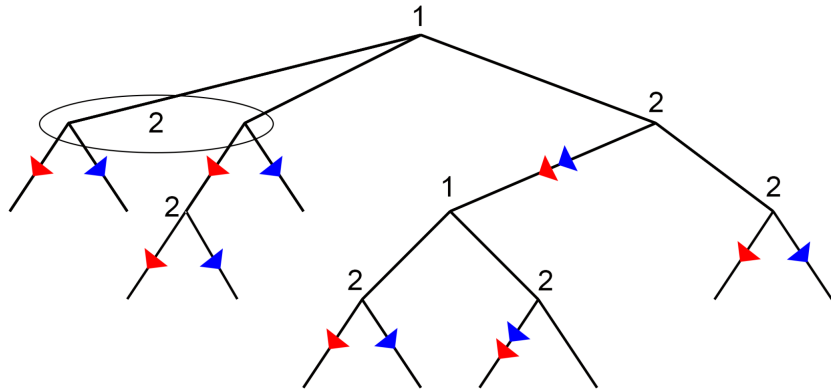


Figure 4

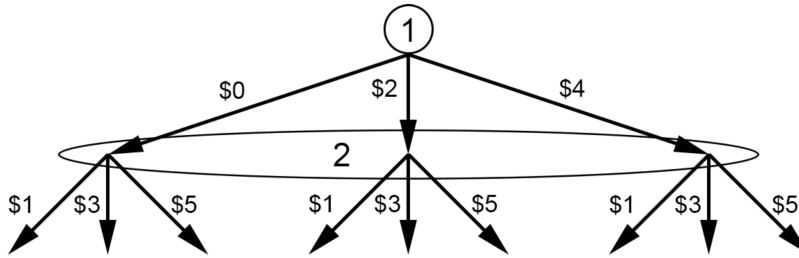


Figure 5

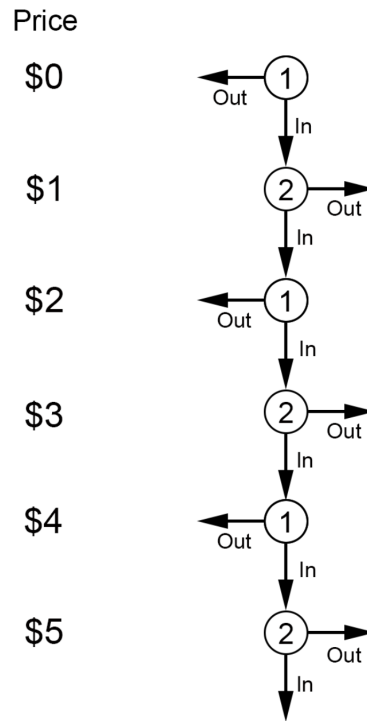


Figure 6

## Acknowledgements

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