

## Interdependent Values II

### Beyond Single-Crossing [1]

What happens when we don't have single-crossing? Can we at least guarantee some approximation to social welfare?

**Example.** [Impossibility for deterministic prior-free mechanisms without SC.]

**Example.** [Impossibility result for randomized mechanisms without SC.]

**A Restricted Class.** Optimal welfare is not attainable for general valuations. For what *natural* restricted class of valuations can we achieve some  $\alpha$ -approximation to optimal social welfare for every profile of signals  $\mathbf{s}$  (prior-free) with an EPIC mechanism?

## Submodularity over Signals [2]

**Definition 1.** Valuation  $v_i(\cdot)$  is *submodular over signals* if, for all  $j$ , when  $\mathbf{s}_{-j}$  is lower,  $v_i(\cdot)$  is more sensitive to  $s_j$ . For all  $j$ , and for any  $\mathbf{s}_{-j} \leq \mathbf{s}'_{-j}$ :

$$\frac{\partial}{\partial s_j} v_i(s_j, \mathbf{s}_{-j}) \geq \frac{\partial}{\partial s_j} v_i(s_j, \mathbf{s}'_{-j})$$

### Random-Sampling Vickrey Auction.

- Elicit  $s_i$  from each bidder  $i$ .
- Assign each bidder into set  $A$  or set  $B$  w.p.  $1/2$  independently.
- For each bidder  $i \in A$ , calculate and use proxy value  $\hat{v}_i = v_i(s_i, \mathbf{0}_{A \setminus i}, \mathbf{s}_B)$ .
- Allocate to the potential winner in  $A$  with the highest proxy value.

**Theorem 1.** *The RS Vickrey Auction is EPIC and achieves a prior-free  $\frac{1}{4}$ -approximation to the optimal welfare.*

To prove this theorem, we need to address (1) truthfulness and (2) the approximation guarantee.

**Truthfulness.** Is this allocation EPIC?

**Approximation.** Is  $v_i(s_i, \mathbf{0}_{A \setminus i}, \mathbf{s}_B)$  a good way to choose a winner?

**Lemma 1** (Key Lemma). *Let  $v_i$  be a submodular over signals valuation. Partition all agents other than  $i$  uniformly at random into sets  $A$  and  $B$ . Then*

$$\mathbb{E}_{A,B}[v_i(s_i, \mathbf{0}_A, \mathbf{s}_B)] \geq \frac{1}{2} v_i(\mathbf{s}).$$

*Proof.*

*Proof of Theorem 1.*

## References

- [1] Alon Eden, Michal Feldman, Amos Fiat, and Kira Goldner. Interdependent values without single-crossing. In *Proceedings of the 2018 ACM Conference on Economics and Computation*, EC '18, pages 369–369, New York, NY, USA, 2018. ACM.
- [2] Alon Eden, Michal Feldman, Amos Fiat, Kira Goldner, and Anna R. Karlin. Combinatorial auctions with interdependent valuations: Sos to the rescue. In *Proceedings of the 2019 ACM Conference on Economics and Computation*, EC '19, Phoenix, AZ, USA, 2019. ACM.