DS 574 Algorithmic Mechanism Design	**Lecture #16 Worksheet**
Fall 2023	Prof. Kira Goldner

Interdependent Values I

Thus far, we have been discussing private independent values. That is, each bidder i has private information $\mathbf{v}_i$ regarding their value for item i.

However, in many settings, the valuations may be correlated between buyers, depend on one another's information, or even be common.

The Interdependent Values Model [2]. Each bidder has a private *signal* s_i that is a piece of information about the item. Each buyer has a **public** *valuation function* $v_i(s_1, \cdots, s_n)$ that dictates how the buyer aggregates the information into a value for the item.

Assumptions on $v_i(\cdot)$:

- $v_i(\cdot)$ monotone in s_j for all i, j.
- $v_i(\cdot)$ is non-negative for all $\mathbf{s}$.

Example: Common Values [4]: The average of estimates $v_i(s_1, \ldots, s_n) = \frac{1}{n}\sum_i s_i \, \forall i$, or the wallet game $v_i(s_1, \ldots, s_n) = \sum_i s_i \, \forall i$.

Optimal Social Welfare

Mechanisms. How can we maximize social welfare in this setting, optimally? What does a mechanism even look like?

- Report:
- Calculate:
- Allocate to:

Incentive Compatibility. What conditions are necessary for maximizing social welfare optimally to be incentive-compatible? What definition of incentive-compatible are we going for?

DSIC? Why or why not?

Next best we can hope for is:

In this context that means:

Definition 1. Truth-telling is said to be [ ] if, for every bidder i, for every possible realization of the other bidders' signals $\mathbf{s}_{-i}$, and given that other bidders report their signals truthfully, then it is in bidder i's best interest to report their true signal.

Myerson in IDV. What is the analogue of Myerson's Lemma in the interdependent setting?

Theorem 1 (Myerson Analogue [3])**.** *Environment:*

(a) An allocation rule $\mathbf{x}$ *is [ ] if and only if*

(b) If $\mathbf{x}$ *is [ ], then there is a unique payment rule such that the sealed-bid mechanism* $(\mathbf{x}, \mathbf{p})$ *is [ ].*

(c) The payment rule in is given by:

$$p_i(\mathbf{s}) = x_i(\mathbf{s})v_i(\mathbf{s}) - \int_{v_i(0,\mathbf{s}_{-i})}^{v_i(s_i,\mathbf{s}_{-i})} x_i(v_i^{-1}(t \mid \mathbf{s}_{-i}), \mathbf{s}_{-i})dt$$

$$- \left[x_i(0,\mathbf{s}_{-i})v_i(0,\mathbf{s}_{-i}) - p_i(0,\mathbf{s}_{-i})\right];$$

$$p_i(0,\mathbf{s}_{-i}) \leq x_i(0,\mathbf{s}_{-i})v_i(0,\mathbf{s}_{-i}).$$

What allocation will maximize social welfare?

Payments. What are the payments?

Truthfulness. Is this mechanism EPIC? When might it not be?

Assumptions. What assumption could we place on the class of valuations to ensure that the mechanism is always EPIC?

Beyond Single-Crossing [1]

What happens when we don't have single-crossing? Can we at least guarantee some approximation to social welfare?

Example. [Impossibility for deterministic prior-free mechanisms without SC.]

Example. [Impossibility result for randomized mechanisms without SC.]

A Restricted Class. Optimal welfare is not attainable for general valuations. For what *natural* restricted class of valuations can we achieve some α-approximation to optimal social welfare for every profile of signals $\mathbf{s}$ (prior-free) with an EPIC mechanism?

References

[1] Alon Eden, Michal Feldman, Amos Fiat, and Kira Goldner. Interdependent values without single-crossing. In *Proceedings of the 2018 ACM Conference on Economics and Computation*, EC '18, pages 369–369, New York, NY, USA, 2018. ACM.

[2] Paul R Milgrom and Robert J Weber. A theory of auctions and competitive bidding. *Econometrica: Journal of the Econometric Society*, pages 1089–1122, 1982.

[3] Tim Roughgarden and Inbal Talgam-Cohen. Optimal and robust mechanism design with interdependent values. *ACM Trans. Econ. Comput.*, 4(3):18:1–18:34, June 2016.

[4] Robert B Wilson. Competitive bidding with asymmetric information. *Management Science*, 13(11):816–820, 1967.