DS 574 Algorithmic Mechanism Design	**Lecture #15 Worksheet**
Fall 2023	Prof. Kira Goldner

Gains from Trade in Two-Sided Markets

Today we study a new setting, the two-sided market setting, and a new objective, gains from trade.

Definition 1. In *two-sided markets*, we have n buyers and m sellers, and a platform facilitating trade. Each seller owns some item(s) and has private values for them $\mathbf{s}_j$ and will not sell below these values. Each buyer has private values for item(s) $\mathbf{b}_i$ and will not buy above these values.

Let's simplify significantly for now and focus on the simplest possible setting: one buyer and one seller for one item. This setting is called *bilateral trade.*

Definition 2. In the *bilateral trade* setting, there is one seller with one item for sale, and item $s \sim F_S$ for their own item. There is also one buyer with item $b \sim F_B$ for the item. The platform's job is to determine a price for the buyer to pay, p^B, and a payment to the seller p^S.

We need to review our standard concepts in this new setting and make sure that we understand them, and see if anything additional is necessary.

Utility.

Budget Balance.

Many Single-Dimensional Buyers and Sellers. Now, we consider the setting with m identical sellers, each seller j with one item and one value $s_j \sim F_S$ for their item. There are n buyers, each with a value b_i for any item, where $b_i \sim F_B$.

Welfare.

Gains from Trade.

OPT vs. Constrained-OPT. Our goal is to maximize GFT, and we would like the mechanism that does so to be

1. Dominant-Strategy Incentive-Compatible
2. Ex-Post Individually Rational
3. Weakly Budget-Balanced

In economics, they call the allocation that is the solution to the unconstrained optimization problem of maximizing GFT "first-best." They call the mechanism that is the solution to the constrained optimization problem of maximizing GFT *subject to* (1-3) "second-best."

Theorem 1 (Myerson Satterthwaite [3])**.** *Even for 1 buyer, 1 seller, and 1 item, the allocation that maximizes GFT (and thus welfare) may not be implementable by any mechanism satisfying (1-3). That is, first-best is not always attainable.*

The Optimal Allocation.

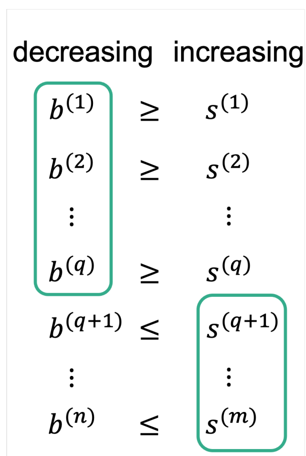

Figure 1: The optimal allocation.

Claim 1. The (post-trade) welfare is equal to the sum of the highest m values in the population.

The Buyer Trade Reduction(BTR) Mechanism [1]. The simple prior-free mechanism we will use is as follows, inspired by McAfee's Trade Reduction mechanism [2]:

1. Solicit all buyer and seller values.
2. Compute the optimal allocation on the reported values.
3. Buy items at some p^S; sell items to buyers at some p^B.

   (a)

   (b)

Let $BTR(n, m)$ denote the GFT from this mechanism in a market with n buyers and m sellers.

Observation 2 (DSIC+IR)**.** *This mechanism is DSIC and ex-post IR because we set prices only using the values of non-winning agents, so winning agents pay prices lower than their values that they cannot impact.*

Observation 3 (Budget Balance)**.** *Setting prices according to (3a) or (3b) satisfies weak budget balance.*

Claim 2. BTR reduces if and only if the $m + 1^{\text{st}}$ highest-valued agent is a seller.

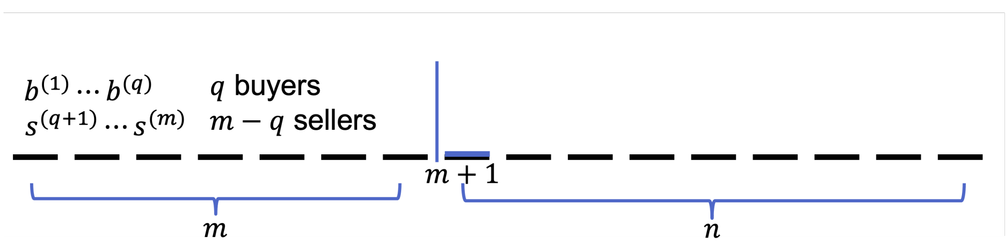

Figure 2: When BTR reduces.

Theorem 4 (Babaioff G. Gonczarowski [1]). *When buyers and sellers are drawn i.i.d. from some distribution F, given an initial market with n buyers and m sellers, running Buyer Trade Reduction on a market with 1 additional buyer yields at least as much GFT as the optimal GFT in the initial market.*

$$BTR(n+1, m) \geq \textsc{opt}(n, m).$$

Proof. Approach: Aim to show that

$$\textsc{opt}(n+1, m) - \textsc{opt}(n, m) \geq \textsc{opt}(n+1, m) - BTR(n+1, m).$$

References

[1] Moshe Babaioff, Kira Goldner, and Yannai A. Gonczarowski. Bulow-klemperer-style results for welfare maximization in two-sided markets. In Shuchi Chawla, editor, *Proceedings of the 2020 ACM-SIAM Symposium on Discrete Algorithms, SODA 2020, Salt Lake City, UT, USA, January 5-8, 2020*, pages 2452–2471. SIAM, 2020.

[2] R Preston McAfee. A dominant strategy double auction. *Journal of economic Theory*, 56(2):434–450, 1992.

[3] Roger B Myerson and Mark A Satterthwaite. Efficient mechanisms for bilateral trading. *Journal of economic theory*, 29(2):265–281, 1983.