DS 574 Algorithmic Mechanism Design	**Lecture #16 Worksheet**
Fall 2022	Prof. Kira Goldner

Robustness: Prior-Independence

"Prior-independent" results give us guarantees in the event that the designer *doesn't* know the distribution F from which the bidders' values are drawn. In this case, we assume that their values are still drawn from a prior distribution, as in the Bayesian setting, so there is some revenue-optimal mechanism $\text{OPT}(F)$ that we wish to approximate, we just have to do so without knowing F.

The Bulow-Klemperer Result

One famous result takes the form of resource augmentation.

Theorem 1 (Bulow Klemperer [1])**.** *For i.i.d. regular single-item environments, the expected revenue of the second-price auction with $n+1$ agents is at least that of the optimal auction with n agents.*

Let's talk about what this theorem is saying. Instead of finding the optimal auction tailored to a distribution F for n agents, you can use the Vickrey auction, which requires no prior knowledge of the distribution, so long as:

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This result **does not hold** without these assumptions. However, it is a *very strong* result, should our setting meet these assumptions.

Proof.

The Single Sample Mechanism

Can't recruit extra buyers? Instead, we can just exclude one. This is what the single sample result says.

Theorem 2 (Dhangwatnotai Roughgarden Yan [2])**.** *Given a random sample from a bidder's distribution, posting it as a take-it-or-leave-it price gives a* $\frac{1}{2}$*-approximation to the optimal revenue.*

Figure 1: Geometric intuition for a posted-price from a single sample.

Proof. In quantile space!

It turns out, using a single sample from the buyers' distribution to set reserve prices and running VCG is a good approximation to the optimal mechanism. See Hartline chapter 5 for more.

Prophet Inequalities

Summary of the setting:

- Goal: Pick one item; maximize its value.
- Gambler knows distribution for each item.
- Order is adversarial.
- Inspect each item online (see v_i) and irrevocably decide whether to take or pass forever.
- Compete with $\textsc{opt} = \mathbb{E}_{\mathbf{v}}[\max_i v_i]$.

The Prophet Inequality problem was posed by Samuel-Cahn '84 [7], with the original solution and analysis that we'll see by Krengel Sucheston '78 [6] and Garling. It was brought to Algorithmic Mechanism Design by Hajiaghayi Kleinberg Sandholm '07 [3], and a new analysis for this case was developed by Kleinberg Weinberg '12 [4, 5].

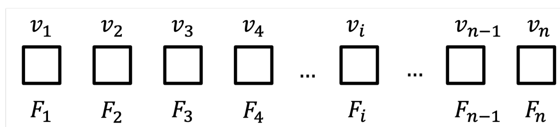

Figure 2: The prophet inequality problem.

Theorem 3. *There is a threshold algorithm* $\textsc{Alg}$ *such that when the gambler takes an item if and only if its value is above* T*,* $\textsc{Alg} \geq \frac{1}{2}\textsc{opt}$*.*

Note: Can you find two different thresholds that give this same approximation?

Proof. We consider two different ways to set the threshold. Let p denote the probability that *some* (at least one) $v_i \geq T$ for $i \in [n]$.

Exercise: You could see this as a mechanism for a buyer to maximize social welfare. Could you design a mechanism to maximize revenue using the prophet inequality?

References

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[4] Robert Kleinberg and S. Matthew Weinberg. Matroid prophet inequalities. In Howard J. Karloff and Toniann Pitassi, editors, *Proceedings of the 44th Symposium on Theory of Computing Conference, STOC 2012, New York, NY, USA, May 19 - 22, 2012*, pages 123–136. ACM, 2012.

[5] Robert Kleinberg and S. Matthew Weinberg. Matroid prophet inequalities and applications to multi-dimensional mechanism design. *Games Econ. Behav.*, 113:97–115, 2019.

[6] Ulrich Krengel and Louis Sucheston. Semiamarts and finite values. *Bulletin of the American Mathematical Society*, 83(4):745–747, 1977.

[7] Ester Samuel-Cahn. Comparison of threshold stop rules and maximum for independent non-negative random variables. *The Annals of Probability*, pages 1213–1216, 1984.