Mechanism Design Basics

Definition 1. Each bidder i has a private *valuation* v_i that is its maximum willingness-to-pay for the item being sold.

Our default assumption is that a bidder's utility is modeled by quasilinear utility.

Definition 2. For a deterministic mechanism with at most one winner, a bidder with *quasilinear utility* has utility

$$u_i(\cdot) = \begin{cases} v_i - p_i & \text{if } i \text{ wins and pays } p_i \\ 0 & \text{otherwise.} \end{cases}$$

Definition 3. A *dominant strategy* is a strategy (bid) that is guaranteed to maximize a bidder's utility *no matter what* the other bidders do.

Sealed-Bid Auctions:

(1) Each bidder i privately communicates a bid b_i to the auctioneer—in a sealed envelope, if you like.

(2) The auctioneer decides who gets the good (if anyone).

(3) The auctioneer decides on a selling price.

How should we do (2) and (3)? For now, (2) will just be giving the item to the highest bidder. What about (3)?

Some potential auctions:

- First-price auction: the price is equal to the highest bid.
- Second-price auction: the price is equal to the second-highest bid.
- All-pay auction: *every bidder* (not just the winning bidder) pays their bid.*

  *Note that we need to amend our definition of quaslinear utility already for the all-pay auction, since we only defined payments in terms of when the bidder wins. For now, we can modify it to

  $$u_i(\cdot) = v_i \cdot \mathbb{1}[i \text{ wins}] - p_i$$

  where p_i is i's assigned payment. In the next class, we'll further modify it.

How should we bid in these auctions? It's not necessarily clear in first-price or all-pay, but it *is* clear in the second-price auction with a bit of reasoning: just bid your true value!

Claim 1 (Dominant-Strategy Incentive Compatibility)**.** In a second-price auction, every bidder has a *dominant strategy*: set its bid b_i equal to its private valuation v_i. That is, this strategy maximizes the utility of bidder i, no matter what the other bidders do.